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**Mathematics: analysis and approaches**  
**Standard level**  
**Paper 2**

Monday 9 May 2022 (morning)

Candidate session number

1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

### Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

**In this question, give all answers correct to two decimal places.**

Sam invests \$ 1700 in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a) Find the amount that Sam will have in his account after 10 years. [3]

David also invests \$ 1700 in a savings account that pays an annual rate of interest of  $r\%$ , compounded yearly. David makes no further payments or withdrawals from this account.

(b) Find the value of  $r$  required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. [2]

(c) Find the interest David will earn over the 10 years. [1]

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2. [Maximum mark: 4]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	Number of students
2	5
3	1
4	4
5	3
6	$x$

The median is 4.5 hours.

(a) Find the value of  $x$ . [2]

(b) Find the standard deviation. [2]

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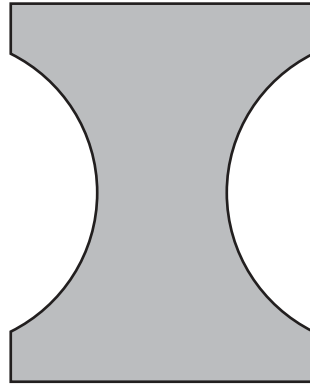
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3. [Maximum mark: 6]

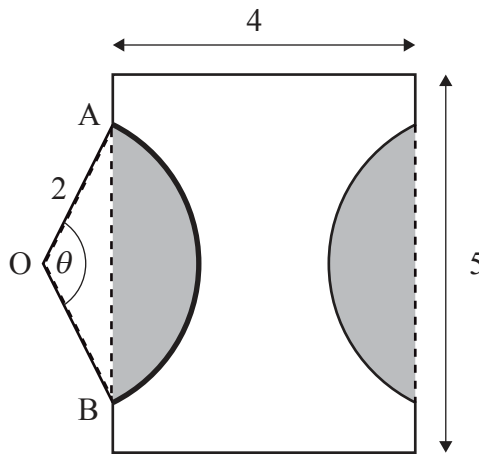
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures 5 cm by 4 cm. The points A and B lie on a circle, with centre O and radius 2 cm, such that  $\widehat{AOB} = \theta$ , where  $0 < \theta < \pi$ . This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of  $\theta$ . [3]

(b) Given that the area of the logo is  $13.4 \text{ cm}^2$ , find the value of  $\theta$ . [3]

(This question continues on the following page)



(Question 3 continued)

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12EP05

Turn over

4. [Maximum mark: 6]

A discrete random variable,  $X$ , has the following probability distribution:

$x$	0	1	2	3
$P(X = x)$	0.41	$k - 0.28$	0.46	$0.29 - 2k^2$

- (a) Show that  $2k^2 - k + 0.12 = 0$ . [1]
- (b) Find the value of  $k$ , giving a reason for your answer. [3]
- (c) Hence, find  $E(X)$ . [2]

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5. [Maximum mark: 7]

A particle moves along a straight line so that its velocity,  $v \text{ m s}^{-1}$ , after  $t$  seconds is given by  $v(t) = e^{\sin t} + 4 \sin t$  for  $0 \leq t \leq 6$ .

- (a) Find the value of  $t$  when the particle is at rest. [2]
- (b) Find the acceleration of the particle when it changes direction. [3]
- (c) Find the total distance travelled by the particle. [2]

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6. [Maximum mark: 6]

Let  $A$  and  $B$  be two independent events such that  $P(A \cap B') = 0.16$  and  $P(A' \cap B) = 0.36$ .

(a) Given that  $P(A \cap B) = x$ , find the value of  $x$ .

[4]

(b) Find  $P(A' | B')$ .

[2]

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12EP08

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### Section B

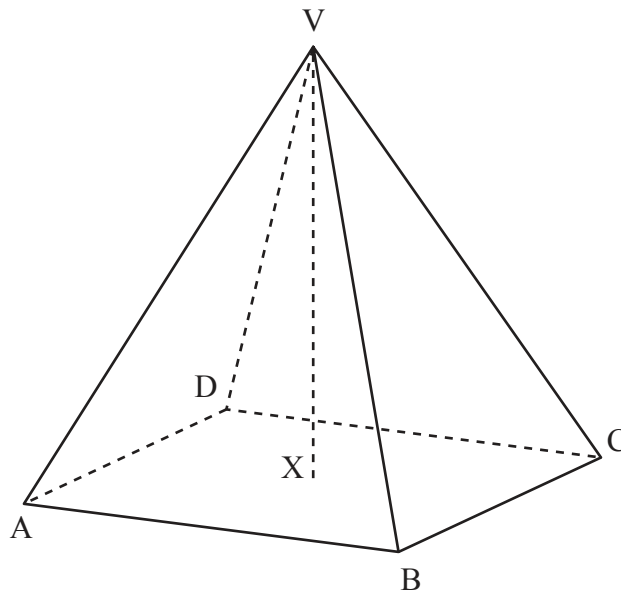
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

**All lengths in this question are in centimetres.**

A solid metal ornament is in the shape of a right pyramid, with vertex  $V$  and square base  $ABCD$ . The centre of the base is  $X$ . Point  $V$  has coordinates  $(1, 5, 0)$  and point  $A$  has coordinates  $(-1, 1, 6)$ .

**diagram not to scale**



(a) Find  $AV$ . [2]

(b) Given that  $\widehat{AVB} = 40^\circ$ , find  $AB$ . [3]

The volume of the pyramid is  $57.2 \text{ cm}^3$ , correct to three significant figures.

(c) Find the height of the pyramid,  $VX$ . [3]

A second ornament is in the shape of a cuboid with a rectangular base of length  $2x \text{ cm}$ , width  $x \text{ cm}$  and height  $y \text{ cm}$ . The cuboid has the same volume as the pyramid.

(d) The cuboid has a minimum surface area of  $S \text{ cm}^2$ . Find the value of  $S$ . [5]



Do **not** write solutions on this page.

8. [Maximum mark: 16]

The function  $f$  is defined by  $f(x) = \frac{4x+1}{x+4}$ , where  $x \in \mathbb{R}$ ,  $x \neq -4$ .

(a) For the graph of  $f$

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

(b) (i) Find  $f^{-1}(x)$ .

(ii) Using an algebraic approach, show that the graph of  $f^{-1}$  is obtained by a reflection of the graph of  $f$  in the  $y$ -axis followed by a reflection in the  $x$ -axis. [8]

The graphs of  $f$  and  $f^{-1}$  intersect at  $x = p$  and  $x = q$ , where  $p < q$ .

(c) (i) Find the value of  $p$  and the value of  $q$ .

(ii) Hence, find the area enclosed by the graph of  $f$  and the graph of  $f^{-1}$ . [5]



Do **not** write solutions on this page

9. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights,  $C$  grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

- (a) Find the probability that a randomly selected chocolate muffin weighs less than 61 g. [2]
- (b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. [2]

The weights,  $B$  grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

- (c) (i) Find the probability that the randomly selected muffin weighs less than 61 g.
- (ii) Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to  $\sigma$  g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

- (d) Find the value of  $\sigma$ . [5]

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**References:**

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12EP11

Please **do not** write on this page.

Answers written on this page  
will not be marked.



12EP12